The features of the unperturbed profile $\bar{u}_z$ also affect the phase velocities of neutral perturbations. For N = 0 for example in the case of constant viscosity, the phase velocity changes sign with a change in the parameters along the neutral curve (Fig. 2) so that the point k = 2.61 (versus k = 2.65 in [7]) corresponds to a neutral "standing" perturbation. With an increase in γ, standing perturbations are shifted into the shortwave region. For example, at $\gamma = 0.5$ and N = 0, k = 2.62. For other values of $N \neq 0$, perturbations with a phase velocity equal to zero cannot exist (Fig. 2), i.e., the perturbations drift along the flow. We might point out the oddness of the profile in the special case N = 1 and $\gamma = 0.5$ [6], and "standing" perturbations are again possible for k = 0.82.

NOTATION

p, convective pressure reckoned from the hydrostatic pressure at the mean density ρ; ν, kinematic viscosity coefficient; β, χ, coefficient of linear expansion and diffusivity, assumed constant; g, acceleration due to gravity; c_p, specific heat.

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DEPENDENCE OF THE MASS TRANSFER DURING DISSOLUTION OF A ROUGH WALL IN A PLANE CHANNEL ON THE STRUCTURE OF THE STREAM

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A channel wall with a sinusoidally rough surface is considered and the location of the point on this surface where the diffusion current reaches its maximum is determined, depending on the Reynolds number as well as on the roughness wave-length and amplitude.

The equation of vortex transport for the flow function ψ and the boundary conditions for steady two-dimensional flow of a viscous incompressible fluid through a plane channel are

$$\frac{\partial^4\psi}{\partial x^4} + 2\frac{\partial^4\psi}{\partial x^2\partial y^2} + \frac{\partial^4\psi}{\partial y^4} = \frac{1}{\nu}\left[\frac{\partial\psi}{\partial y}\frac{\partial\Delta\psi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\Delta\psi}{\partial y}\right], \tag{1}$$

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$$y = 0 \quad \frac{\partial\psi}{\partial x} = 0; \quad y = \pm h_1 \quad \frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial y} = 0. \tag{2}$$

Here Δ is the Laplace operator and $h_1 = h_0 + A\cos\frac{2\pi x}{\lambda}$ is the channel half width. Equation (1) can be reduced to dimensionless form by introduction of the dimensionless coordinates $\eta = y/A$ and $\xi = x/\lambda$, with small parameters $\varepsilon = A/\lambda$ and $\delta = A/h_0$, and the dimensionless flow function $\Psi = \psi(U_{max}A)$. With the solution sought in the form of the series

$$\Psi = \sum_{k=0}^{\infty} \varepsilon^k \Psi_k(\xi, \eta, R, \delta) \tag{3}$$

as proposed in another study [2], it is possible to extract from Eq. (1) terms of the same power in k and to form dimensionless equations describing, respectively, zeroth-order, first-order, second-order, etc. flow

$$\frac{\partial^4\Psi_0}{\partial\eta^4} = 0, \tag{4}$$

$$\frac{\partial^4\Psi_1}{\partial\eta^4} = R\left(\frac{\partial\Psi_0}{\partial\eta}\frac{\partial^3\Psi_0}{\partial\xi\partial\eta^2} - \frac{\partial\Psi_0}{\partial\xi}\frac{\partial^3\Psi_0}{\partial\eta^3}\right), \tag{5}$$

$$\frac{\partial^4\Psi_2}{\partial\eta^4} = -2\frac{\partial^4\Psi_0}{\partial\xi^2\partial\eta^2} + R\left(\frac{\partial\Psi_0}{\partial\eta}\frac{\partial^3\Psi_1}{\partial\xi\partial\eta^2} + \frac{\partial\Psi_1}{\partial\eta}\frac{\partial^3\Psi_0}{\partial\xi\partial\eta^2} - \frac{\partial\Psi_0}{\partial\xi}\frac{\partial^3\Psi_1}{\partial\eta^3} - \frac{\partial\Psi_1}{\partial\xi}\frac{\partial^3\Psi_0}{\partial\eta^3}\right), \tag{6}$$

$$\frac{\partial^4\Psi_3}{\partial\eta^4} = -2\frac{\partial^4\Psi_1}{\partial\xi^2\partial\eta^2} + R\left(\frac{\partial\Psi_0}{\partial\eta}\frac{\partial^3\Psi_2}{\partial\xi\partial\eta^2} + \frac{\partial\Psi_1}{\partial\eta}\frac{\partial^3\Psi_1}{\partial\xi\partial\eta^2} + \right.$$
$$\left. + \frac{\partial\Psi_2}{\partial\eta}\frac{\partial^3\Psi_0}{\partial\xi\partial\eta^2} - \frac{\partial\Psi_0}{\partial\xi}\frac{\partial^3\Psi_2}{\partial\eta^3} - \frac{\partial\Psi_1}{\partial\xi}\frac{\partial^3\Psi_1}{\partial\eta^3} - \frac{\partial\Psi_2}{\partial\xi}\frac{\partial^3\Psi_0}{\partial\eta^3}\right) + R\left(\frac{\partial\Psi_0}{\partial\eta}\frac{\partial^3\Psi_0}{\partial\xi^3} - \frac{\partial\Psi_0}{\partial\xi}\frac{\partial^3\Psi_0}{\partial\xi^2\partial\eta}\right) \tag{7}$$

with the boundary conditions

$$\eta = 0 \quad \frac{\partial\Psi_0}{\partial\xi} = \frac{\partial\Psi_1}{\partial\xi} = \ldots = 0, \quad \frac{\partial^2\Psi_0}{\partial\eta^2} = \frac{\partial^2\Psi_1}{\partial\eta^2} = \ldots = 0,$$
$$\Psi_0 = -\frac{2}{3}, \quad \Psi_1 = \Psi_2 = \ldots = 0,$$
$$\eta = h \quad \frac{\partial\Psi_0}{\partial\eta} = \frac{\partial\Psi_1}{\partial\eta} = \ldots = 0, \quad \frac{\partial\Psi_0}{\partial\xi} = \frac{\partial\Psi_1}{\partial\xi} = \ldots = 0,$$
$$\Psi_0 = \Psi_1 = \ldots = 0, \tag{8}$$

where the Reynolds number is $N_{Re} = U_{max}A/\nu$ and $h = h/A$. Upon changing to the variables $\xi_1 = \xi$ and $Y = \eta/h$, Eqs. (4)-(7) can be integrated and their solution obtained in the form (3)

$$\Psi \doteq Y - \frac{Y^3}{3} - \frac{2}{3} + Y(1-Y^2)^2 \sum_{k=1}^{\infty}\sum_{n=1}^{2k}\sum_{m=0}^{n-1} \varepsilon^k A_{2k,n}(n-m)\,Y^{2m}, \tag{9}$$

where the coefficients $A_{2k,n}$ are functions of ξ_1:

$$A_{21} = \frac{Rh'}{30}; \quad A_{22} = -\frac{Rh'}{210}; \quad A_{41} = -0.1\left(\frac{h'}{h}\right)^{\bullet} h^2 + 0.3\left(\frac{h'}{h}\right)^2 h^2 +$$
$$+ \frac{Rh}{105}\left[-A_{21}' + 19\left(\frac{h'}{h}\right)A_{21}\right]; \quad A_{42} = \frac{Rh}{735}\left[23A_{21}' - 68\left(\frac{h'}{h}\right)A_{21}\right];$$
$$A_{43} = \frac{Rh}{120}\left[-7A_{21}' + 3\left(\frac{h'}{h}\right)A_{21}\right]; \quad A_{44} = \frac{Rh}{6930}\left[5A_{21}' - 14\left(\frac{h'}{h}\right)A_{21}\right];$$

$$A_{61} = \frac{1}{120}\left\{\frac{h^2}{7}\left[132 A_{21}'' - 792\left(\frac{h'}{h}\right)A_{21}' - 396\left(\frac{h'}{h}\right)' A_{21} + \right.\right.$$

$$\left. + 1188\left(\frac{h'}{h}\right)^2 A_{21}\right] + Rh^3\left[-\left(\frac{h'}{h}\right)'' + 2\left(\frac{h'}{h}\right)'\left(\frac{h'}{h}\right)\right] + Rh\left[-10A_{41}' - 14A_{42}' - 18A_{43}' - 22A_{44}' + \left(\frac{h'}{h}\right)\times\right.$$

$$\left.\left.\times\left(28A_{41} + 44A_{42} + 60A_{43} + 76A_{44} + \frac{660}{49}A_{21}^2\right)\right]\right\};$$

$$A_{62} = \frac{1}{840}\left\{h^2\left[-40A_{21}'' + A_{21}\left(400\frac{h'}{h} + 200\left(\frac{h'}{h}\right)' - \right.\right.\right.$$

$$\left.\left. - 1000\left(\frac{h'}{h}\right)^2\right)\right] + Rh^3\left[2\left(\frac{h'}{h}\right)'' - 8\left(\frac{h'}{h}\right)'\left(\frac{h}{h}\right)\right] + Rh\left[28A_{41}' + 12A_{42}' + 16A_{43}' + 20A_{44}' - \left(\frac{h'}{h}\right)\times\right.$$

$$\left.\left.\times\left(48A_{41} + 72A_{42} + 96A_{43} + 120A_{44} + \frac{5756}{49}A_{21}^2\right) + \frac{52}{49}A_{21}A_{21}'\right]\right\};$$

$$A_{63} = \frac{1}{3024}\left\{h^2\left[12A_{21}'' + A_{21}\left(-168\left(\frac{h'}{h}\right) - 84\left(\frac{h'}{h}\right)' + 588\left(\frac{h'}{h}\right)^2\right)\right] + Rh^3\left[-\left(\frac{h'}{h}\right)'' + 6\left(\frac{h'}{h}\right)'\left(\frac{h'}{h}\right)\right] + Rh\left[42A_{42}' - \right.\right.$$

$$\left.\left. - 18A_{41}' + \left(\frac{h'}{h}\right)\left(-84A_{42} + 60A_{41} + \frac{2040}{7}A_{21}^2\right) - \frac{144}{7}A_{21}A_{21}'\right]\right\};$$

$$A_{64} = \frac{1}{7920}\left[72A_{43}' - 40A_{42}' + \left(\frac{h'}{h}\right)\left(-144A_{43} + 122A_{43} - \frac{1928}{7}A_{21}^2\right) + \frac{1432}{49}A_{21}A_{21}'\right],$$

$$A_{65} = \frac{1}{17166}\left[110A_{44}' - 70A_{43}' + \left(\frac{h'}{h}\right)(-220A_{44} + 180A_{43} + 100A_{21}^2) - \frac{80}{7}A_{21}A_{21}'\right];$$

$$A_{66} = \frac{1}{32766}\left[-108A_{44}' + \left(\frac{h'}{h}\right)(264A_{44} - 12A_{21}^2) + \frac{12}{7}A_{21}'A_{21}\right];$$

. . . .

Here the prime sign denotes differentiation with respect to ξ_1.

In the region adjacent to the wall, where $Y \simeq 1$, the flow function Ψ can be expressed as

$$\Psi = -(1-Y)^2(1-\alpha), \tag{10}$$

where

$$\alpha = 4\sum_{k=1}^{\infty}\sum_{n=1}^{2k}\sum_{m=0}^{n-1}\varepsilon^k A_{2k,n}(n-m).$$

This solution can be used for determining the components of the stream velocity $u = \frac{\partial\psi}{\partial y}$, and $v = -\frac{\partial\psi}{\partial x}$ along the areas x and y, respectively. In the dimensionless form these velocity components are

$$U = \frac{1}{h}\frac{\partial\psi}{\partial Y}, \quad V = -\varepsilon\left(\frac{\partial\Psi}{\partial\xi_1} - \frac{h'}{h}Y\frac{\partial\Psi}{\partial Y}\right),$$

or, taking into account relation (10), they are described by the equalities

$$U\Big|_Y = \frac{2}{h}(1-Y)(1-\alpha), \tag{11}$$

$$V\Big|_Y = 2\varepsilon\frac{h'}{h}(1-Y)(1-\alpha). \tag{12}$$

In the cases of either separation flow or nonseparation flow one can, on the basis of the Reynolds analogy and its equivalent in terms of forces [3], expect the diffusion current with a constant dynamic viscosity to be maximum at the point of maximum internal friction. This point is determined by the value of $\frac{dU_s}{dn}\Big|_{Y=1}$, with U_s denoting the tangential component

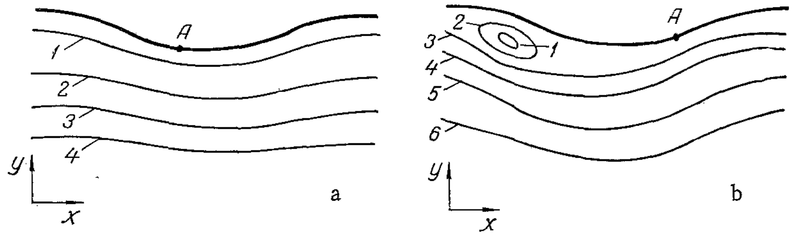

Fig. 1. Streamline pattern Ψ at a rough wall: a) δ = 0.1; ε = 0.01; N_{Re} = 25; 1) Ψ = −0.01; 2) −0.1; 3) −0.2; 4) −0.5; b) δ = 0.1; ε = 0.01; N_{Re} = 110; 1) Ψ = 0.003; 2) 0.0003; 3) −0.0003; 4) −0.01; 5) −0.1; 6) −0.5.

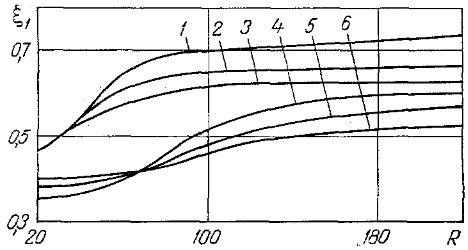

Fig. 2. Dependence of the location of the point on the channel surface with the maximum diffusion current on the Reynolds number, at various values of ε and δ: 1) δ = 0.01, ε = 0.002; 2) 0.05 and 0.01; 3) 0.1 and 0.02; 4) 0.01 and 0.001; 5) 0.01 and 0.005; 6) 0.1 and 0.01.

of velocity along the wall contour, referred to U_{max}, and n is the normal to the wall referred to A. With U_s expressed as

$$U_s = U\cos\Theta + V\sin\Theta,$$

where Θ is the inclination angle of the wall, relations (11) and (12) yield for $\left.\frac{dU_s}{dn}\right|_{Y=1}$

$$\left.\frac{dU_s}{dn}\right|_{Y=1} = \frac{2}{h^2}\,(1+\varepsilon^2 h'^2)(1-\alpha). \tag{13}$$

From expressions (13) one can determine the conditions under which the surface roughness of the channel wall will decrease. When the frictional stress is maximum at ξ_1 = 0.5, then dissolution of the material of convexities will predominate and the amplitude of roughness will decrease. When the frictional stress is maximum at $\xi_1 \neq 0.5$, then the roughness profile will change.

An evaluation of the function $\left.\frac{dU_s}{dn}\right|_{Y=1}$ with the aid of a Nairi-K computer, according to relation (13), has revealed that with δ = 0.1, ε = 0.01, and N_{Re} = 20 there takes place a laminar streamlining of the asperities and the diffusion current is maximum at the point ξ_1 = 0.45 (point A in Fig. 1a). As the Reynolds number increases to 60, the streamlining remains laminar but the point of maximum diffusion current shifts to ξ_1 = 0.50 now. As the Reynolds number increases further, up to the limit imposed by the requirement that series (3) remains a convergent one, there occurs a separation of the stream and a vortex forms whose dimensions increase while the point of maximum diffusion current shifts toward ξ_1 = 1. The streamline pattern with N_{Re} = 110, e.g., is shown in Fig. 1b. Here the diffusion current is maximum at the point ξ_1 = 0.65 (point A in Fig. 1b). Formation of a vortex depends largely on the value of ε. As this parameter increases, a vortex forms and the point of maximum diffusion current shifts toward $\xi_1 = 1$ at lower values of the Reynolds number (Fig. 2). The surface smoothing factor $c=\left(\frac{dU_s}{dn}\Big|_{Y=1}\right)_{\max}\Big/\left(\frac{dU_s}{dn}\Big|_{Y=1}\right)_{\min}$ increases with higher values of the Reynolds number: $c = 1.2$ when $N_{Re} = 20$ and $c = 2.1$ when $N_{Re} = 100$.

Therefore, a dissolving surface is smoothed most intensely prior to formation of a vortex in a surface cavity. The roughness amplitude changes during dissolution and, therefore, the optimum conditions for smoothing will be ensured only by maintenance of a continuously vortex-free mode of streamlining.

The results of this study can be utilized in setting up the finish treatment in chemical manufacturing processes, in electrochemical polishing, and in other technologies where formation of the roughness microprofile of surfaces is largely influenced by the hydrodynamics of their streamlining.

NOTATION

x, longitudinal coordinate; y, normal coordinate; ξ, η, Y, dimensionless coordinates; h_0, mean channel half-width; h_1, channel half-width; h, dimensionless channel half-width; λ, roughness wavelength; A, roughness amplitude; ψ, flow function; Ψ, dimensionless flow function; U_{max}, maximum velocity in the channel; u, v, u_s, velocity components along the x axis, the y axis, and along the wall contour, respectively; U, V, U_s, dimensionless velocity components; ν, kinematic viscosity; $A_{2k,n}$, coefficients of the series expansion; N_{Re}, Reynolds number; c, surface smoothing factor; and ε, δ, parameters.

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GROWTH OF PARAFFIN DEPOSITS ON THE PIPE SURFACE IN A STREAM OF OIL

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In describing the kinetics of the growth of a layer, it was taken that the solidification temperature is a variable and depends on the paraffin concentration in the stream of oil in a pipe.

When the soil temperature in the vicinity of a pipe transporting paraffinous crude drops, the layers of the liquid near the pipe surface solidify and lose their mobility. These phenomena are connected with the appearance of paraffin crystals in the near-wall layers of the liquid; these crystals form the structure within which the liquid is retarded. The temperature at which the near-wall layers of the liquid lose their mobility is called the solidification point. Experiments showed that the solidification point depends on the paraffin concentration in the stream [1]. This circumstance must be taken into account in describing the kinetics of the growth of the layer of deposits on the inner pipe surface.

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